International Journal of Solids and Structures 41 (2004) 5231–5246

www.elsevier.com/locate/ijsolstr

Dynamic responses of piezoelectric hollow cylinders in an axial magnetic field

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Received 5 August 2003; received in revised form 2 April 2004
Available online 6 May 2004

Abstract

This paper presents an analytical solution for the interaction of electric potentials, electric displacements, elastic deformations and mechanical loads, and describes electromagnetoelastic responses and perturbation of the magnetic field vector in a piezoelectric hollow cylinder subjected to sudden mechanical load and electric potential. An interpolation method is applied to solve the Volterra integral equation of the second kind caused by interactions between different physical fields. By means of finite integral transforms, Laplace transforms, and their inverse transforms, the exact expressions for the dynamic responses of stresses, electric displacements, electric potentials and perturbation response of an axial magnetic field vector in the piezoelectric hollow cylinders are obtained. The present method is suitable for piezoelectric hollow cylinders in an axial magnetic field, subjected to arbitrary mechanical loads and electrical potential shocks. Finally, numerical results are carried out and discussed.

Keywords: Piezoelectric structure; Orthotropic hollow cylinder; Electromagnetoelastic dynamic; Perturbation of magnetic field vector

1. Introduction

The interaction of electric potentials, electric displacements and elastic deformations in structures is studied due to its many engineering applications in the fields of magnetic storage elements, magnetic structural elements, plasma physics and the corresponding measurement techniques of magnetoelasticity. The coupling of elastic deformation, electric field and magnetic field gives rise to the theory of dynamic coupled electromagnetomechanics, which is known to be especially suitable in the high frequency and short wave-length modes described by Eringen (2003). Shul'ga et al. (1984) investigated the axisymmetric electroelastic waves in a hollow piezoelectric ceramic cylinder by using a method based on representation of the solution in the form of powers of the radial coordinate, and gave an analysis of dispersion relations and the

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doi:10.1016/j.ijsolstr.2004.04.019

Nomenclature

$\vec{U}$, u_r	displacement vector and radial displacement [m]
c_{ij}, e_{ij}, ε_{ij}	elastic constants [N/m^2], piezoelectric constants [C/m^2] and dielectric constants [C^2/N m^2]
σ_{ij}, D_{rr}	the component of stresses [N/m^2] and radial electric displacement [C/m^2]
$\psi(r,t)$	electric potential [W/A]
f_{zz}	Lorentz's force [kg/m^2 s^2]
ρ, t	mass density [kg/m^3] and time [s]
r	radius [m]
a, b	inner and outer radii of piezoelectric hollow cylinder [m]
μ	magnetic permeability [H/m]
$\vec{H}$	magnetic intensity vector $(0,0,H_z)$
$\vec{h}$	perturbation of magnetic field vector $(0,0,h_z)$
$\vec{J}$	electric current density vector
$\vec{e}$	perturbation of electric field vector
C_L	electromagnetoelastic wave speed [m/s]
ω	the inherent frequency of the hollow cylinder [1/s].

effect of the piezoelectric effect on dynamic characteristics of the waveguide. Chand et al. (1990) presented the investigations of the distribution of deformation, temperature, stresses and magnetic field in a homogeneous isotropic, thermally and electrically conducting, uniformly rotating elastic half-space, in contact with vacuum, due to impulsive load at the plane boundary, utilizing the generalized theory of thermoelasticity. Dhaliwal and Saxena (1991) applied generalized elasticity theory to solve the problem of magnetothermoelastic waves produced by thermal shock in an infinite elastic solid with a cylindrical cavity, obtained approximate small-time solutions in some cases, and gave numerical results for displacement, temperature, and stresses. Sherief and Ezzat (1996) used the Laplace transform technique to find the distribution of thermal stresses and temperature in a generally thermoelastic and electrically conducting half-space under sudden thermal shock and permeated by a primarily uniform magnetic field. Solodyak and Gachkevich (1996) presented an analytical method for obtaining electromagnetic and temperature fields as well as mechanical stresses in a ferromagnetic solid subjected to a harmonic electromagnetic field at the frequencies usually used in industry. The free vibrations of piezoelectric, empty and also compressible fluid filled cylindrical shells as three-dimensional problems were studied by Ding et al. (1997) using triangle series. Ezzat (1997) described the distribution of thermal stresses and temperature in a perfectly conducting half-space when suddenly heated to a constant temperature, and compared that with the results obtained in the absence of a magnetic field by using the method of potentials and Laplace transform techniques. The technique of finite integral transforms is presented in Wang and Lu (2002) to analyse magnetothermoelastic waves and perturbation of the magnetic field vector produced by thermal shock in a solid conducting cylinder, and the focusing effect on both magnetothermostress and perturbation of the axial magnetic field vector was revealed in the paper. By virtue of the variable separation technique and the interpolation method, Ding et al. (2003) investigated the axisymmetric plane strain electroelastic dynamic problem of a hollow cylinder, and presented the numerical results for the displacements, stresses, electric displacements and electric potentials. To date, investigations on the dynamic response of coupled fields have mostly considered magnetothermoelasticity and electroelasticity. The investigations on the dynamic responses of the coupling of three physical fields for electricity, magnet and elastic deformation have been limited because of its complexity.

In the paper, an analytical method is developed for solving the dynamic responses of piezoelectric hollow cylinders in an axial magnetic field, subjected to sudden mechanical load and electric potential shock. First, the electromagnetodynamic equilibrium equation for an orthotropic piezoelectric hollow cylinder is derived. Second, the electromagnetodynamic equilibrium equation is decomposed into a homogeneous quasi-static equation, which satisfies the inhomogeneous boundary conditions, and an inhomogeneous dynamic solution, which satisfies homogeneous boundary conditions. By using the method described in Lekhniskii (1981), the solution for the homogeneous quasi-static equation which satisfies the inhomogeneous boundary conditions is obtained. After using an interpolation method to solve the Volterra integral equation of the second kind caused by interactions, the solution for the inhomogeneous dynamic solution which satisfies homogeneous boundary conditions can be obtained by means of the finite Hankel transforms (Cinelli, 1965), the Laplace transforms, and their inverse transforms. Thus, the solution for the dynamic responses of piezoelectric hollow cylinders, in an axial magnetic field, subjected to sudden mechanical load and electric potential is rigorously derived.

Finally, some practical examples are calculated. The histories and distributions of the dynamic stresses, the electric displacement, the electric potential and the perturbation of magnetic field vector are carried out. The feature of the solution is related to the propagation of the cylindrical wave. The interactions among the dynamic stress, the electric displacement, the electric potential and the perturbation of magnetic field vector are carried out and discussed.

2. Statement of the problem and basic formulations

A long, piezoelectric hollow cylinder placed initially in an axial magnetic field $\vec{H}(0,0,H_z)$ is shown in Fig. 1. For the axisymmetic plane strain problem, the components of displacement and electric potential in the cylindrical coordinate (r,θ,z) system are expressed as $u_\theta=0$, $u_z=0$, $u_r=u_r(r,t)$ and $\psi=\psi(r,t)$,

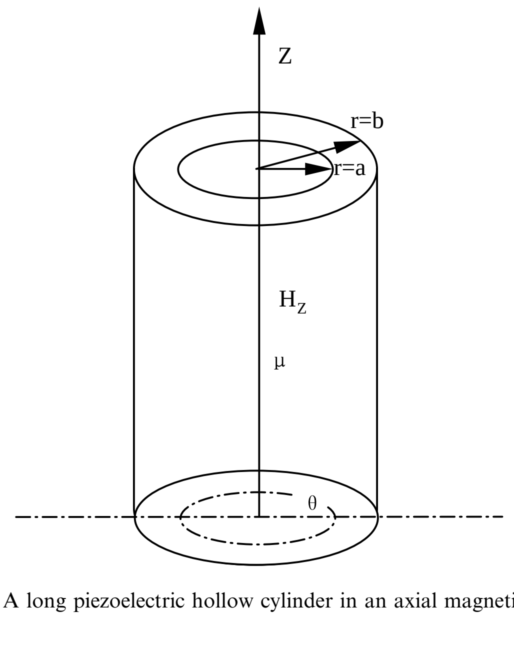

Fig. 1. A long piezoelectric hollow cylinder in an axial magnetic field.

respectively. The constitutive equations of orthotropic, radially polarized piezoelectric material are expressed as

$$\sigma_{rr} = c_{rr}\frac{\partial u_r}{\partial r} + c_{r\theta}\frac{u_r}{r} + e_{rr}\frac{\partial \psi}{\partial r} \tag{1a}$$

$$\sigma_{\theta\theta} = c_{r\theta}\frac{\partial u_r}{\partial r} + c_{\theta\theta}\frac{u_r}{r} + e_{r\theta}\frac{\partial \psi}{\partial r} \tag{1b}$$

$$\sigma_{zz} = c_{rz}\frac{\partial u_r}{\partial r} + c_{\theta z}\frac{u_r}{r} + e_{rz}\frac{\partial \psi}{\partial r} \tag{1c}$$

$$D_{rr} = e_{rr}\frac{\partial u_r}{\partial r} + e_{r\theta}\frac{u_r}{r} - \varepsilon_{rr}\frac{\partial \psi}{\partial r} \tag{1d}$$

where c_{ij}, e_{ij} and ε_{ij} are elastic constants, piezoelectric constants and dielectric constants in cylindrical coordinate (r, θ, z) system, respectively. σ_{ij} and D_{rr} are the components of stress and radial electric displacement, respectively.

The boundary conditions are

$$\sigma_{rr}(a,t) = P_{a0}(t) \quad \sigma_{rr}(b,t) = P_{b0}(t) \tag{2a}$$

$$\psi(a,t) = \psi_a(t) \quad \psi(b,t) = \psi_b(t) \tag{2b}$$

Assuming that the magnetic permeability, μ, (Ezzat, 1997) of the piezoelectric hollow cylinder equals the magnetic permeability of the medium around it, the governing electrodynamic Maxwell equations (John, 1984) are given by

$$\overline{J} = \nabla \times \bar{h}, \quad \nabla \times \bar{e} = -\mu\frac{\partial \bar{h}}{\partial t}, \quad \mathrm{div}\,\bar{h} = 0,$$

$$\bar{e} = -\mu\left(\overline{U}\frac{\partial \overline{U}}{\partial t} \times \overline{H}\right) \quad \bar{h} = \nabla \times (\overline{U} \times \overline{H}) \tag{3}$$

Applying an initial magnetic field vector $\vec{H}(0,0,H_z)$ in cylindrical coordinate (r, θ, z) system to Eq. (3), yields

$$\overline{U} = (u_r(r,t), 0, 0), \quad \bar{e} = -\mu\left(0, H_z\frac{\partial u_r}{\partial t}, 0\right), \tag{4a}$$

$$\bar{h} = (0,0,h_z), \quad \overline{J} = \left(0, -\frac{\partial h_z}{\partial r}, 0\right), \quad h_z = -H_z\left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r}\right) \tag{4b}$$

The electomagnetic dynamic equilibrium equation of the piezoelectric hollow cylinder is expressed as

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} + f_{zz} = \rho\frac{\partial^2 u_r}{\partial t^2} \tag{5}$$

where ρ is the mass density, f_{zz} is defined as Lorentz's force (John, 1984), which can be written as

$$f_{zz} = \mu(\overline{J} \times \overline{H}) = \mu H_z^2\frac{\partial}{\partial r}\left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r}\right) \tag{6}$$

In order to simplify calculation, the non-dimensional forms are given by

$$
\begin{aligned}
&c_1=\frac{c_{r\theta}}{c_{rr}},\quad c_2=\frac{c_{\theta\theta}}{c_{rr}},\quad c_3=\frac{c_{rz}}{c_{rr}},\quad c_4=\frac{c_{\theta z}}{c_{rr}},\quad e_1=\frac{e_{rr}}{\sqrt{c_{rr}\varepsilon_{rr}}}\\
&e_2=\frac{e_{r\theta}}{\sqrt{c_{rr}\varepsilon_{rr}}},\quad e_3=\frac{e_{rz}}{\sqrt{c_{rr}\varepsilon_{rr}}},\quad \sigma_i=\frac{\sigma_{ii}}{c_{rr}}(i=r,\theta,z),\quad \phi=\sqrt{\frac{\varepsilon_{rr}}{c_{rr}}}\frac{\psi}{b}\\
&D_r=\frac{D_{rr}}{\sqrt{c_{rr}\varepsilon_{rr}}},\quad u=\frac{u_r}{b},\quad \xi=\frac{r}{b},\quad s=\frac{a}{b},\quad C_{\mathrm{V}}=\sqrt{\frac{c_{rr}}{\rho}},\quad \tau=\frac{C_{\mathrm{V}}t}{b},\quad f_z=\frac{f_{zz}}{c_{rr}}b\\
&P_a(\tau)=\frac{P_{a0}(t)}{c_{rr}},\quad P_b(\tau)=\frac{P_{b0}(t)}{c_{rr}},\quad \phi_a=\sqrt{\frac{\varepsilon_{rr}}{c_{rr}}}\frac{\psi_a}{b},\quad \phi_b=\sqrt{\frac{\varepsilon_{rr}}{c_{rr}}}\frac{\psi_b}{b}
\end{aligned}
\tag{7}
$$

Then, Eqs. (1) and (5) can be rewritten as follows:

$$\sigma_r=\frac{\partial u}{\partial\xi}+c_1\frac{u}{\xi}+e_1\frac{\partial\phi}{\partial\xi}\tag{8a}$$

$$\sigma_\theta=c_1\frac{\partial u}{\partial\xi}+c_2\frac{u}{\xi}+e_2\frac{\partial\phi}{\partial\xi}\tag{8b}$$

$$\sigma_z=c_3\frac{\partial u}{\partial\xi}+c_4\frac{u}{\xi}+e_3\frac{\partial\phi}{\partial\xi}\tag{8c}$$

$$D_r=e_1\frac{\partial u}{\partial\xi}+e_2\frac{u}{\xi}-\frac{\partial\phi}{\partial\xi}\tag{8d}$$

$$\frac{\partial\sigma_r}{\partial\xi}+\frac{\sigma_r-\sigma_\theta}{\xi}+f_z=\frac{\partial^2u}{\partial\tau^2}\tag{8e}$$

$$f_z=\frac{\mu H_z^2}{c_{rr}}\frac{\partial}{\partial\xi}\left(\frac{\partial u}{\partial\xi}+\frac{u}{\xi}\right)\tag{8f}$$

In absence of free charge density, the charge equation of electrostatics (Heyliger, 1996) is expressed as

$$\frac{1}{\xi}\frac{\partial}{\partial\xi}(\xi D_r(\xi,\tau))=0\tag{9}$$

Solving Eq. (9), gives

$$D_r(\xi,\tau)=\frac{1}{\xi}d(\tau)\tag{10}$$

The corresponding boundary conditions and the initial conditions are expressed as

$$\sigma_r(s,\tau)=P_a(\tau)\quad \sigma_r(1,\tau)=P_b(\tau)\tag{11a}$$

$$\phi(s,\tau)=\phi_a(\tau)\quad \phi(1,\tau)=\phi_b(\tau)\tag{11b}$$

$$\tau=0\quad u(\xi,0)=u_0(\xi)\quad \dot{u}(\xi,\tau)=v_0(\xi)\tag{11c}$$

3. Solution of the problem

Substituting Eq. (10) into Eq. (8d), yields

$$\frac{\partial \phi}{\partial \xi} = e_1 \frac{\partial u}{\partial \xi} + e_2 \frac{u}{\xi} - \frac{1}{\xi} d(\tau) \tag{12}$$

Substituting Eq. (12) into Eqs. (8a) and (8b), gives

$$\sigma_r = (1 + e_1^2) \frac{\partial u}{\partial \xi} + (c_1 + e_1 e_2) \frac{u}{\xi} - \frac{e_1}{\xi} d(\tau) \tag{13a}$$

$$\sigma_\theta = (c_1 + e_1 e_2) \frac{\partial u}{\partial \xi} + (c_2 + e_2^2) \frac{u}{\xi} - \frac{e_2}{\xi} d(\tau) \tag{13b}$$

Substituting Eq. (13) into Eq. (8e), the electromagnetodisplacement equilibrium equation is expressed as

$$\frac{\partial^2 u(\xi,\tau)}{\partial \xi^2} + \frac{1}{\xi} \frac{\partial u(\xi,\tau)}{\partial \xi} - \frac{H^2 u(\xi,\tau)}{\xi^2} = \frac{1}{C_{\mathrm{L}}^2} \frac{\partial^2 u(\xi,\tau)}{\partial \tau^2} + m \frac{d(\tau)}{\xi^2} \tag{14}$$

where

$$H^2 = \frac{c_{rr}(c_2 + e_2^2) + \mu H_z^2}{c_{rr}(1 + e_1^2) + \mu H_z^2}, \quad C_{\mathrm{L}}^2 = \frac{c_{rr} + c_{rr} e_1^2 + \mu H_z^2}{c_{rr}}, \quad m = -\frac{c_{rr} e_2}{c_{rr}(1 + e_1^2) + \mu H_z^2} \tag{15}$$

Substituting Eq. (13a) into Eq. (11a), the corresponding stress boundary condition can be written as

$$\xi = s: \quad \frac{\partial u(\xi,\tau)}{\partial \xi} + h \frac{u(\xi,\tau)}{\xi} = P_1(\tau) \tag{16a}$$

$$\xi = 1: \quad \frac{\partial u(\xi,\tau)}{\partial \xi} + h \frac{u(\xi,\tau)}{\xi} = P_2(\tau) \tag{16b}$$

where

$$h = \frac{c_1 + e_1 e_2}{1 + e_1^2} \quad P_1(\tau) = \frac{1}{1 + e_1^2} \left[P_a(\tau) + \frac{e_1}{s} d(\tau) \right] \quad P_2(\tau) = \frac{1}{1 + e_1^2} [P_b(\tau) + e_1 d(\tau)] \tag{17}$$

Assume that the general solution of the basic equation (14) is of the form (Eringen and Suhubi, 1975)

$$u(\xi,\tau) = u_{\mathrm{s}}(\xi,\tau) + u_{\mathrm{d}}(\xi,\tau) \tag{18}$$

where $u_{\mathrm{s}}(\xi,\tau)$ and $u_{\mathrm{d}}(\xi,\tau)$ are the quasi-static solution and the dynamic solution of Eq. (14), respectively.

The quasi-static solution $u_{\mathrm{s}}(\xi,\tau)$ must satisfy the following equation and the corresponding boundary condition:

$$\frac{\partial^2 u_{\mathrm{s}}(\xi,\tau)}{\partial \xi^2} + \frac{1}{\xi} \frac{\partial u_{\mathrm{s}}(\xi,\tau)}{\partial \xi} - \frac{H^2 u_{\mathrm{s}}(\xi,\tau)}{\xi^2} = m \frac{d(\tau)}{\xi^2} \tag{19a}$$

$$\left[\frac{\partial u_{\mathrm{s}}(\xi,\tau)}{\partial \xi} + h \frac{u_{\mathrm{s}}(\xi,\tau)}{\xi} \right]_{\xi = s} = P_1(\tau) \tag{19b}$$

$$\left[\frac{\partial u_{\mathrm{s}}(\xi,\tau)}{\partial \xi} + h \frac{u_{\mathrm{s}}(\xi,\tau)}{\xi} \right]_{\xi = 1} = P_2(\tau) \tag{19c}$$

Using the method described in Lekhniskii (1981), the general solution of Eq. (19a) is expressed as

$$u_{\rm s}(\xi,\tau)=B_1\xi^H+\frac{B_2}{\xi^H}-\frac{md(\tau)}{H^2} \tag{20}$$

where B_1 and B_2 are unknown constants which can be determined by making use of the boundary conditions (19b) and (19c). Thus, Eq. (20) is rewritten as

$$u_{\rm s}(\xi,\tau)=\varphi_1(\xi)P_a(\tau)+\varphi_2(\xi)P_b(\tau)+\varphi_3(\xi)d(\tau) \tag{21}$$

where

$$\varphi_1(\xi)=\frac{g_1}{1+e_1^2}\xi^H+\frac{g_2}{1+e_1^2}\xi^{-H} \tag{22a}$$

$$\varphi_2(\xi)=-\frac{g_1 s^{-(H+1)}}{1+e_1^2}\xi^H-\frac{g_2 s^{(H-1)}}{1+e_1^2}\xi^{-H} \tag{22b}$$

$$\varphi_3(\xi)=\left[\frac{e_1}{(1+e_1^2)}+\frac{hm}{H^2}\right]\left[\left(\frac{1}{s}-s^{-(H+1)}\right)g_1\xi^H+\left(\frac{1}{s}-s^{(H-1)}\right)g_2\xi^{-H}\right]-\frac{m}{H^2} \tag{22c}$$

$$g_1=\frac{1}{(H+h)[s^{H-1}-s^{-(H+1)}]},\quad g_2=\frac{1}{(H-h)[s^{H-1}-s^{-(H+1)}]} \tag{22e}$$

Substituting Eq. (18) into Eq. (14), and utilizing Eq. (19), the dynamic solution $u_{\rm d}(\xi,\tau)$ should satisfy the following inhomogeneous equation (23), the corresponding homogeneous boundary conditions (24) and the initial conditions (25)

$$\frac{\partial^2 u_{\rm d}(\xi,\tau)}{\partial\xi^2}+\frac{1}{\xi}\frac{\partial u_{\rm d}(\xi,\tau)}{\partial\xi}-\frac{H^2}{\xi^2}u_{\rm d}(\xi,\tau)=\frac{1}{C_{\rm L}^2}\left[\frac{\partial^2 u_{\rm d}(\xi,\tau)}{\partial\tau^2}+\frac{\partial^2 u_{\rm s}(\xi,\tau)}{\partial\tau^2}\right] \tag{23}$$

$$\left[\frac{\partial u_{\rm d}(\xi,\tau)}{\partial\xi}+h\frac{u_{\rm d}(\xi,\tau)}{\xi}\right]_{\xi=s}=0\quad\left[\frac{\partial u_{\rm d}(\xi,\tau)}{\partial\xi}+h\frac{u_{\rm d}(\xi,\tau)}{\xi}\right]_{\xi=1}=0 \tag{24}$$

$$u_{\rm d}(\xi,0)+u_{\rm s}(\xi,0)=u_0\quad\frac{\partial u_{\rm d}(\xi,0)}{\partial\tau}+\frac{\partial u_{\rm s}(\xi,0)}{\partial\tau}=v_0 \tag{25}$$

In Eq. (23), $u_{\rm s}(\xi,\tau)$ is the known solution as shown in Eq. (21).

Utilizing Eq. (24), the eigen-equation of the homogeneous form (let $u_{\rm s}=0$) of Eq. (23) is expressed as

$$J_aY_b-J_bY_a=0, \tag{26}$$

where

$$\begin{aligned}J_a&=k_iJ'_{\rm H}(k_is)+h\frac{J_{\rm H}(k_is)}{s}\quad Y_a=k_iY'_{\rm H}(k_is)+h\frac{Y_{\rm H}(k_is)}{s}\\J_b&=k_iJ'_{\rm H}(k_i)+hJ_{\rm H}(k_i)\quad Y_b=k_iY'_{\rm H}(k_i)+hY_{\rm H}(k_i)\end{aligned} \tag{27a–d}$$

$J_{\rm H}(k_i\xi)$ and $Y_{\rm H}(k_i\xi)$ are the first and the second kind of the Hth-order Bessel function, respectively. In these expressions, $k_i(i=1,2,\ldots,n)$ denotes a series of positive roots for eigen-equation (26). The natural frequencies are

$$\omega_i=C_{\rm L}k_i \tag{28}$$

From Cinelli (1965), defining $\bar{u}_{\rm d}(k_i,\tau)$ as the finite Hankel transform of $u_{\rm d}(\xi,\tau)$, yields

$$\bar{u}_{\rm d}(k_i,\tau)=H[u_{\rm d}(\xi,\tau)]=\int_s^1\xi u_{\rm d}(\xi,\tau)C_{\rm H}(k_i\xi)\,{\rm d}\xi \tag{29}$$

The inverse transform to Eq. (29) is defined as

$$u_{\rm d}(\xi,\tau)=\sum_{k_i}\frac{\bar{u}_{\rm d}(k_i,\tau)}{F(k_i)}C_{\rm H}(k_i\xi) \tag{30a}$$

where

$$C_{\rm H}(k_i\xi)=J_{\rm H}(k_i\xi)Y_a-J_aY_{\rm H}(k_i\xi) \tag{30b}$$

$$\begin{aligned}F(k_i)&=\int_s^1\xi[C_{\rm H}(k_i\xi)]^2\,{\rm d}\xi\\&=\frac{J_a^2}{J_b^2}\frac{2}{k_i^2\pi^2}\left\{h^2+k_i^2\left[1-\left(\frac{H}{k_i}\right)^2\right]\right\}-\frac{2}{k_i^2\pi^2}\left\{\left(\frac{h}{s}\right)^2+k_i^2\left[1-\left(\frac{H}{k_is}\right)^2\right]\right\}\end{aligned} \tag{30c}$$

Applying the finite Hankel transform (29) to (23), and utilizing the boundary conditions (24), yield

$$-k_i^2\bar{u}_{\rm d}(k_i,\tau)=\frac{1}{C_{\rm L}^2}\left[\frac{\partial^2\overline{u_{\rm d}}(k_i,\tau)}{\partial\tau^2}+\frac{\partial^2\overline{u_{\rm s}}(k_i,\tau)}{\partial\tau^2}\right] \tag{31}$$

where $\bar{u}_{\rm s}(k_i,\tau)=H[u_{\rm s}(\xi,\tau)]$.

Applying the Laplace transforms for Eq. (31), gives

$$\overline{u_{\rm d}}^*(k_i,p)=-\overline{u_{\rm s}}^*(k_i,p)+\frac{\omega_i^2}{(\omega_i^2+p^2)}\overline{u_{\rm s}}^*(k_i,p)+\frac{p\overline{u_0}(k_i)}{(\omega_i^2+p^2)}+\frac{\overline{v_0}(k_i)}{(\omega_i^2+p^2)} \tag{32}$$

where p is the Laplace transform parameter, and $\overline{u_0}(k_i)=H[\overline{u_0}(\xi)]$, $\overline{v_0}(k_i)=H[\overline{v_0}(\xi)]$. The inverse transform of Eq. (32) is given by

$$\overline{u_{\rm d}}(k_i,\tau)=\bar{\varphi}_1(k_i)I_{1i}(k_i,\tau)+\bar{\varphi}_2(k_i)I_{2i}(k_i,\tau)+\bar{\varphi}_3(k_i)I_{3i}(k_i,\tau)+I_{4i}(k_i,\tau) \tag{33}$$

where $\bar{\varphi}_1=H[\varphi_1]$, $\bar{\varphi}_2=H[\varphi_2]$, $\bar{\varphi}_3=H[\varphi_3]$

$$\begin{aligned}I_{1i}(k_i,\tau)&=-P_a(\tau)+\omega_i\int_0^\tau P_a(t)\sin[\omega_i(\tau-t)]\,{\rm d}t\\I_{2i}(k_i,\tau)&=-P_b(\tau)+\omega_i\int_0^\tau P_b(t)\sin[\omega_i(\tau-t)]\,{\rm d}t\\I_{3i}(k_i,\tau)&=-d(\tau)+\omega_i\int_0^\tau d(t)\sin[\omega_i(\tau-t)]\,{\rm d}t\\I_{4i}(k_i,\tau)&=\overline{u_0}(k_i)\cos(\omega_i\tau)+\overline{v_0}(k_i)\frac{1}{\omega_i}\sin(\omega_i\tau)\end{aligned} \tag{34}$$

Substituting Eq. (33) into Eq. (30), gives

$$u_{\rm d}(\xi,\tau)=\sum_{k_i}\frac{C_{\rm H}(k_i\xi)}{F(k_i)}[\bar{\varphi}_1(k_i)I_{1i}(k_i,\tau)+\bar{\varphi}_2(k_i)I_{2i}(k_i,\tau)+\bar{\varphi}_3(k_i)I_{3i}(k_i,\tau)+I_{4i}(k_i,\tau)] \tag{35}$$

Substituting Eqs. (21) and (35) into Eq. (18), the solution of the dynamic responses of piezoelectric hollow cylinders is expressed as

$$u(\xi,\tau)=\varphi_1(\xi)P_a(\tau)+\varphi_2(\xi)P_b(\tau)+\varphi_3(\xi)d(\tau)+\sum_{k_i}\frac{C_{\mathrm{H}}(k_i\xi)}{F(k_i)}[\bar{\varphi}_1(k_i)I_{1i}(k_i,\tau)+\bar{\varphi}_2(k_i)I_{2i}(k_i,\tau)+\bar{\varphi}_3(k_i)I_{3i}(k_i,\tau)+I_{4i}(k_i,\tau)] \tag{36}$$

It is noted that in the above expression, $d(\tau)$ is an unknown function which is related to the electric displacement. It is necessary to determine $d(\tau)$ in the following. Integrating Eq. (12) and utilizing the corresponding electric boundary condition (11b), yield

$$\phi(\xi,\tau)=\Phi_1(\xi)P_a(\tau)+\Phi_2(\xi)P_b(\tau)+\Phi_3(\xi)d(\tau)+\sum_i\Phi_{4i}(\xi)F_i(\tau)+\phi_a(\tau) \tag{37}$$

where

$$\Phi_1(\xi)=e_1\left[\varphi_1(\xi)-\varphi_1(s)-\sum_{k_i}\frac{(C_{\mathrm{H}}(k_i\xi)-C_{\mathrm{H}}(k_is))}{F(k_i)}\bar{\varphi}_1(k_i)\right]+e_2\int_s^\xi\frac{1}{\zeta}\left[\varphi_1(\zeta)-\sum_{k_i}\frac{C_{\mathrm{H}}(k_i\zeta)}{F(k_i)}\bar{\varphi}_1(k_i)\right]\mathrm{d}\zeta \tag{38a}$$

$$\Phi_2(\xi)=e_1\left[\varphi_2(\xi)-\varphi_2(s)-\sum_{k_i}\frac{(C_{\mathrm{H}}(k_i\xi)-C_{\mathrm{H}}(k_is))}{F(k_i)}\bar{\varphi}_2(k_i)\right]+e_2\int_s^\xi\frac{1}{\zeta}\left[\varphi_2(\zeta)-\sum_{k_i}\frac{C_{\mathrm{H}}(k_i\zeta)}{F(k_i)}\bar{\varphi}_2(k_i)\right]\mathrm{d}\zeta \tag{38b}$$

$$\Phi_3(\xi)=e_1\left[\varphi_3(\xi)-\varphi_3(s)-\sum_{k_i}\frac{(C_{\mathrm{H}}(k_i\xi)-C_{\mathrm{H}}(k_is))}{F(k_i)}\bar{\varphi}_3(k_i)\right]+e_2\int_s^\xi\frac{1}{\zeta}\left[\varphi_3(\zeta)-\sum_{k_i}\frac{C_{\mathrm{H}}(k_i\zeta)}{F(k_i)}\bar{\varphi}_3(k_i)\right]\mathrm{d}\zeta-\ln\left(\frac{\xi}{s}\right) \tag{38c}$$

$$\Phi_{4i}(\xi)=e_1\frac{(C_{\mathrm{H}}(k_i\xi)-C_{\mathrm{H}}(k_is))}{F(k_i)}+e_2\int_s^\xi\frac{1}{\zeta}\frac{C_{\mathrm{H}}(k_i\zeta)}{F(k_i)}\mathrm{d}\zeta \tag{38d}$$

and

$$F_i(\tau)=F_{1i}(\tau)+\bar{\varphi}_3(k_i)\omega_i\int_0^\tau d(t)\sin[\omega_i(\tau-t)]\,\mathrm{d}t \tag{39a}$$

$$F_{1i}(\tau)=\bar{\varphi}_1(k_i)\omega_i\int_0^\tau P_a(t)\sin[\omega_i(\tau-t)]\,\mathrm{d}t+\bar{\varphi}_2(k_i)\omega_i\int_0^\tau P_b(t)\sin[\omega_i(\tau-t)]\,\mathrm{d}t+\overline{u_0}(k_i)\cos(\omega_i\tau)+\overline{v_0}(k_i)\frac{1}{\omega_i}\sin(\omega_i\tau) \tag{39b}$$

When $\xi=1$, Eq. (37) can be rewritten as

$$\phi_b(\tau)=\Phi_1(1)P_a(\tau)+\Phi_2(1)P_b(\tau)+\Phi_3(1)d(\tau)+\sum_i\Phi_{4i}(1)F_i(\tau)+\phi_a(\tau) \tag{40}$$

Substituting $\tau=0$ into Eq. (40), yields

$$d(0)=\frac{\phi_b(0)-\phi_a(0)-\Phi_1(1)P_a(0)-\Phi_2(1)P_b(0)-\sum_i\Phi_{4i}(1)F_i(0)}{\Phi_3(1)} \tag{41}$$

Substituting Eq. (40) into Eq. (39a), gives

$$\vartheta(\tau) = M_1 d(\tau) + \sum_i M_{2i} \int_0^\tau d(t) \sin[\omega_i(\tau - t)]\,\mathrm{d}t \tag{42a}$$

where

$$\begin{aligned} &\vartheta(\tau) = \phi_b(\tau) - \phi_a(\tau) - \Phi_1(1)P_a(\tau) - \Phi_2(1)P_b(\tau) - \sum_i \Phi_{4i}(1)F_{1i}(\tau) \\ &M_1 = \Phi_3(1), \quad M_{2i} = \Phi_{4i}(1)\bar{\varphi}_3(k_i)\omega_i \end{aligned} \tag{42b}$$

It is seen that Eq. (42a) is Volterra integral equation of the second kind (Kress, 1989). In the following, Eq. (42) is solved by using the recursion formula based on linear interpolation function. Dividing the time interval $[0, \tau]$ into n subintervals, the discrete time points are $\tau_0 = 0, \tau_1, \tau_2, \ldots, \tau_n$. The interpolation function at the time interval $[\tau_{j-1}, \tau_j]$ is expressed as

$$d(\tau) = \xi_j(\tau)d(\tau_{j-1}) + \eta_j(\tau)d(\tau_j) \quad (j = 1, 2, \ldots, n) \tag{43}$$

where

$$\xi_j(\tau) = \frac{\tau - \tau_j}{\tau_{j-1} - \tau_j}, \quad \eta_j(\tau) = \frac{\tau - \tau_{j-1}}{\tau_j - \tau_{j-1}} \quad (j = 1, 2, \ldots, n) \tag{44}$$

Substituting Eq. (43) into Eq. (42a), gives

$$\vartheta(\tau_j) = M_1 d(\tau_j) + \sum_i M_{2i} \sum_{k=1}^{j} [R_{ijk} d(\tau_{k-1}) + S_{ijk} d(\tau_k)] \tag{45}$$

where

$$\begin{aligned} R_{ijk} &= \int_{\tau_{k-1}}^{\tau_k} \xi_k(t) \sin[\omega_i(\tau - t)]\,\mathrm{d}t \\ S_{ijk} &= \int_{\tau_{k-1}}^{\tau_k} \eta_k(t) \sin[\omega_i(\tau - t)]\,\mathrm{d}t \quad (k = 1, 2, \ldots, j, \ j = 1, 2, \ldots, n) \end{aligned} \tag{46}$$

Solving Eq. (45), gives

$$d(\tau_j) = \frac{\vartheta(\tau_j) - \sum_i M_{2i} \sum_{k=1}^{j-1} [R_{ijk} d(\tau_{k-1}) + S_{ijk} d(\tau_k)] - d(\tau_{j-1}) \sum_i M_{2i} R_{ijj}}{M_1 + \sum_i M_{2i} S_{ijj}} \quad (j = 1, 2, \ldots, n) \tag{47}$$

Substituting $d(0)$ in Eq. (41) into Eq. (47), $d(\tau_j)$, $(j = 1, 2, \ldots, n)$ can be determined step by step. Thus, the exact expression of the dynamic displacement $u(\xi, \tau)$ is obtained. The dynamic stresses $\sigma_r(\xi, \tau)$, $\sigma_\theta(\xi, \tau)$, the dynamic electric displacement $D_r(\xi, \tau)$, the dynamic electric potential $\phi(\xi, \tau)$, and the perturbation of magnetic field vector $h_z(\xi, \tau)$ are easily obtained from Eqs. (36), (37), (8) and (4b).

4. Numerical results and discussions

The dynamic responses of piezoelectric hollow cylinders in an axial magnetic field subjected to a sudden pressure on the internal surface and a sudden electric potential on the external surface are considered. In the numerical calculations, the material constants for the piezoelectric hollow cylinder are taken as : $c_{rr} = c_{zz} = 110.0$ GPa, $c_{r\theta} = 77.8$ GPa, $c_{rz} = c_{\theta z} = 115.0$ GPa, $c_{\theta\theta} = 220.0$ GPa, $e_{rz} = e_{r\theta} = -5.2$ (C/m^2), $e_{rr} = 15.1$ (C/m^2), $\varepsilon_{rr} = 5.62 \times 10^{-9}$ (C^2/N m^2) and $\rho = 4350$ (kg/m^3), the internal radius of the piezoelectric hollow cylinder is taken as $a = 0.01$ m, and an equal time step is used to obtain the simplest recursion formula determining the coupling function $d(\tau)$ in the Volterra integral equation of the second kind (42a). In order to ensure the precision of the result, the equal time step $\Delta\tau = 0.01$ is taken. When series terms $k_i = 50$, the relative errors of solutions are less than 1%.

Example 1. The dynamic responses of the piezoelectric hollow cylinder in an axial magnetic field subjected to only a sudden pressure on the internal surface are considered. The corresponding boundary conditions are expressed as

$$\sigma_r(s,\tau) = -\delta(\tau) \quad \sigma_r(1,\tau) = 0 \tag{48a}$$

$$\phi(s,\tau) = 0 \quad \phi(1,\tau) = 0 \tag{48b}$$

where $\delta(\tau)$ expresses the Heaviside function.

(a) A special case in which the ratio of internal radius to external radius $s = a/b = 1/20$, and the dimensionless response time $\tau 1 = \frac{C_L\tau}{sC_V} = \frac{C_L t}{a}$ is taken. When the computing time $\tau 1 \leqslant 20$, it is before the wavefront of responded waves arrives at the external boundary $R = [\frac{r-a}{a}]_{r=b} = 20$, and reflected waves have not been produced at the external boundary. In the above case, the histories of radial stress, hoop stress and perturbation of magnetic field vector at $r = a, 2a$ and $3a$ are, respectively, shown in Fig. 2a–c. The curves in Fig. 2a–c clearly show the features of the compression waves propagating in the piezoelectric hollow cylinders subjected to sudden internal pressure and an axial magnetic field. From Fig. 2a it is seen that the radial stresses at $r = a$ and $r = 21a$ are, respectively, equal to -1 and zero, which satisfies the internal and external boundary conditions (48a). From Fig. 2a–e it is seen that the dynamic responses and perturbation of magnetic field vector at some points equal zero before the arrival of the wavefront, and have strong discontinuities at the points where the wavefront arrives at. The amplitude of the wavefront decays gradually, and the dynamic response approaches to the solution of quasi-static equation at the same point when time is large and the effects of reflected waves have not been produced. Due to the effects of the strong discontinuities, the sign of the hoop stress at the wavefront is reversed as compared to that of the quasi-static hoop stress as shown in Fig. 2b. Fig. 2d and e show the response histories and distributions of the electric displacements $D_r(\xi,\tau)$ and the electric potentials $\phi(\xi,\tau)$ at the different radial points in the piezoelectric hollow cylinder subjected to a suddenly pressure on the internal surface. From Fig. 2d and e, it is easily seen that the response histories and distributions of the electric displacements $D_r(\xi,\tau)$ and the electric potential $\phi(\xi,\tau)$ are similar to that of the dynamic stresses as shown in Fig. 2a and b. They will also arrive finally at a steady value when time τ is large and the effects of reflected waves have not been produced. From Fig. 2e it is also seen that the electric potentials $\phi(\xi,\tau)$ at the internal and external boundary equals zero, which satisfies the prescribed electric boundary conditions (48b). The above descriptions show that the solution in the paper possesses wave properties, and the correctness of the numerical results is validated.

(b) In the following calculation, the ratio of internal radius to external radius is taken as $s = a/b = 1/2$, the dimensionless time $\tau 1 = \frac{C_L\tau}{(1-s)C_V} = \frac{C_L t}{b-a}$, the dimensionless radial coordinate $R = \frac{\xi - s}{1-s} = \frac{r-a}{b-a}$. When the computing time $\tau 1 \leqslant 20$, because of the small wall thickness, $a/b = 1/2$, when the responded time is taken as $\tau 1 \geqslant 1$, the effects of wave reflected between the inner-wall and outer-wall have been produced. From Fig. 3a–e, it is shown that, except that the radial stresses and electric potentials at the internal and external surfaces in the piezoelectric hollow cylinder satisfy the given boundary condition, the dynamic stresses, the electric displacement, the electric potential, and the perturbation of magnetic field vector at other points oscillate dramatically because of the effect of wave reflected between the inner wall and outer wall. From Fig. 3b–d, it is shown that the peak values of hoop stresses, perturbation of magnetic field vector and electric displacements decrease gradually from the inner-wall to the outer-wall at the same time τ.

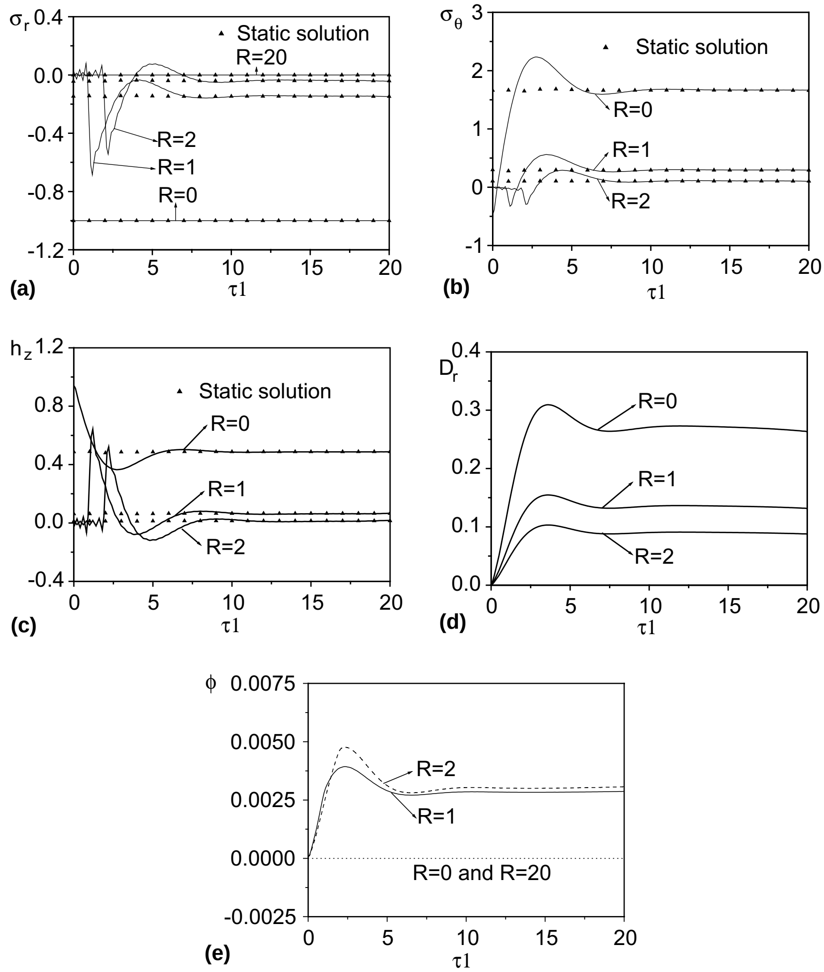

Fig. 2. Response histories of: (a) dynamic stress σ_r at $R = 0$, $R = 1$, $R = 2$ and $R = 20$, where $R = (r - a)/a$, $\tau 1 = C_L t/a$; (b) dynamic stress σ_θ at $R = 0$, $R = 1$, and $R = 2$, where $R = (r - a)/a$, $\tau 1 = C_L t/a$; (c) perturbation of magnetic field vector h_z at $R = 0$, $R = 1$ and $R = 2$, where $R = (r - a)/a$, $\tau 1 = C_L t/a$; (d) dynamic electric displacement D_r at $R = 0$, $R = 1$ and $R = 2$, where $R = (r - a)/a$, $\tau 1 = C_L t/a$ and (e) dynamic electric displacement ϕ at $R = 0$, $R = 1$ $R = 2$ and $R = 20$, where $R = (r - a)/a$, $\tau 1 = C_L t/a$.

Example 2. The dynamic responses of the piezoelectric hollow cylinder in an axial magnetic field subjected to only a sudden electric potential on the external surface are considered in the following calculation. The corresponding boundary conditions are written as

$$\sigma_r(s, \tau) = 0 \quad \sigma_r(1, \tau) = 0 \tag{49a}$$

$$\phi(s, \tau) = 0 \quad \phi(1, \tau) = \delta(\tau) \tag{49b}$$

The ratio of internal radius to external radius is taken as $s = a/b = 1/2$, the dimensionless time $\tau 1 = \frac{C_L \tau}{(1-s)C_V} = \frac{C_L t}{b-a}$, the dimensionless radial coordinate $R = \frac{\xi - s}{1-s} = \frac{r-a}{b-a}$. When the responded time is taken as $\tau 1 \geqslant 1$, because of the small wall thickness, $a/b = 1/2$, and the effects of wave reflected between the inner-wall and outer-wall have been produced. From Fig. 4a and e, it is seen that the radial stresses and the

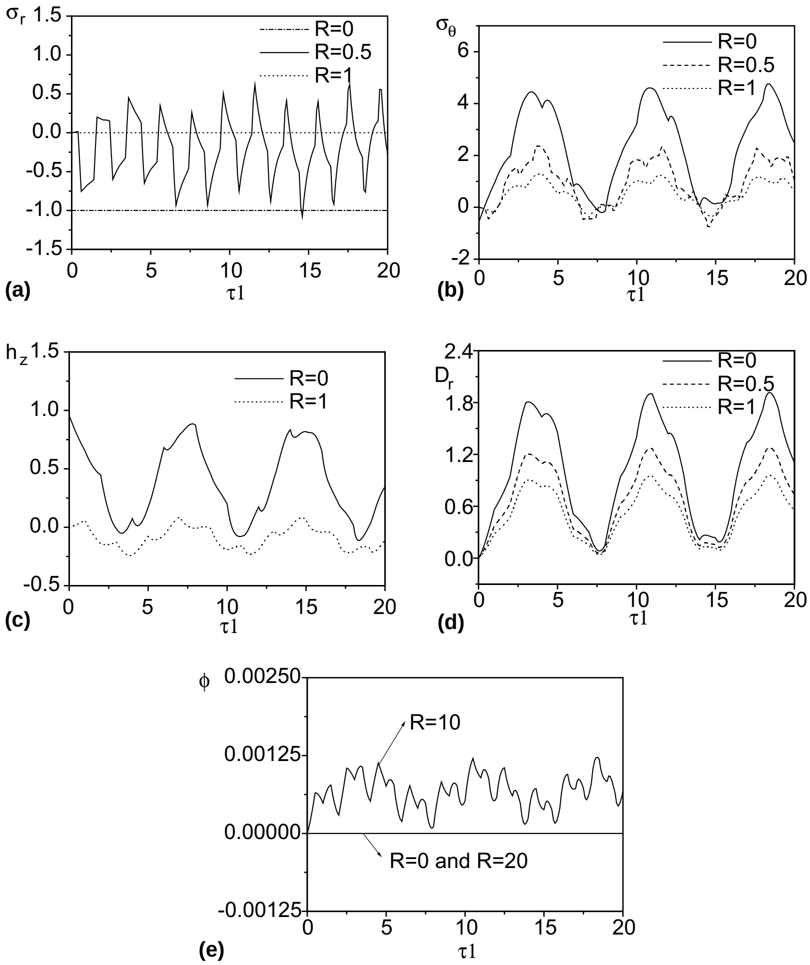

Fig. 3. Response histories of: (a) dynamic stress σ_r at $R = 0$, $R = 0.5$ and $R = 1$, where $R = (r - a)/(b - a)$, $\tau 1 = C_{\mathrm{L}}t/(b - a)$; (b) dynamic stress σ_θ at $R = 0$, $R = 0.5$ and $R = 1$, where $R = (r - a)/(b - a)$, $\tau 1 = C_{\mathrm{L}}t/(b - a)$; (c) perturbation of magnetic field vector h_z, at $R = 0$ and $R = 1$, where $R = (r - a)/(b - a)$, $\tau 1 = C_{\mathrm{L}}t/(b - a)$; (d) dynamic electric displacement D_r, at $R = 0$, $R = 0.5$ and $R = 1$, where $R = (r - a)/(b - a)$, $\tau 1 = C_{\mathrm{L}}t/(b - a)$; (e) dynamic electric potential ϕ, at $R = 0$, $R = 0.5$ and $R = 1$, where $R = (r - a)/(b - a)$, $\tau 1 = C_{\mathrm{L}}t/(b - a)$.

electric potential at the boundaries $R = 0, 1$ satisfy the given boundary conditions. Fig. 4a–e show that except the points at given boundary conditions, all dynamic responses at other points oscillate dramatically around the corresponding quasi-static values because of the effects of wave reflected between the inner-wall and outer-wall. Fig. 4b shows that the amplitude of the hoop compression stress in the piezoelectric hollow cylinder subjected to only unit electric potential at outer wall is larger than the amplitude of the hoop tensile stress. Fig. 4c depicts the response of the perturbation of magnetic field vector at $R = 0$ and $R = 1$. From the curves in Fig. 4c, it is seen that because of the effects of wave reflected the variation of the perturbation of magnetic field vector is similar to that in Fig. 3c. Comparing Figs. 3c and 4c, it is seen that the respondent amplitude of the perturbation of magnetic field vector caused by the sudden unit electric potential is larger than that caused by the sudden unit pressure. Fig. 4d shows that the electric displacement value is negative in the cylinder, and the corresponding absolute value decreases with the increasing of R.

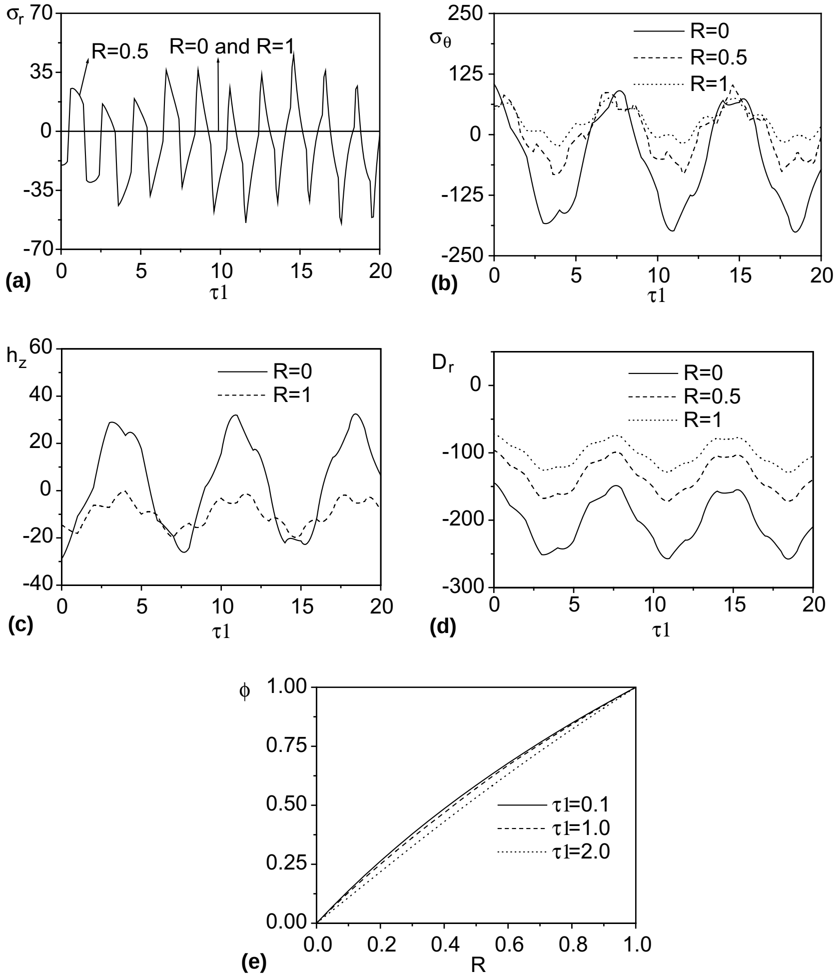

Fig. 4. Response histories of: (a) dynamic stress σ_r at $R = 0$, $R = 0.5$ and $R = 1$, where $R = (r - a)/(b - a)$, $\tau 1 = C_L t/(b - a)$; (b) dynamic stress σ_θ at $R = 0$, $R = 0.5$ and $R = 1$, where $R = (r - a)/(b - a)$, $\tau 1 = C_L t/(b - a)$; (c) perturbation of magnetic field vector h_z, at $R = 0$ and $R = 1$, where $R = (r - a)/(b - a)$, $\tau 1 = C_L t/(b - a)$; and (d) dynamic electric displacement D_r, at $R = 0$, $R = 0.5$ and $R = 1$, where $R = (r - a)/(b - a)$, $\tau 1 = C_L t/(b - a)$. (e) Distributions of dynamic electric potential ϕ, at $\tau = 0.1$, $\tau = 1.0$ and $\tau = 2.0$, where $R = (r - a)/(b - a)$, $\tau 1 = C_L t/(b - a)$.

Fig. 4e shows the distribution of electric potential in the piezoelectric hollow cylinder subjected to sudden unit electric potential at outer wall, which is apparently different from that in the piezoelectric hollow cylinder subjected to sudden unit pressure as shown in Fig. 3e. From Fig. 4e it is seen that the distribution of electric potential along the radial direction of the cylinder is weakly non-linear, and it changes as the respondent time changes.

Example 3. In order to prove further the correctness of analytical results in the paper, omitting the axial magnetic field load in Eq. (5), the present method can be applied to solve the transient problem of piezoelectric hollow cylinders. For ease of comparison with reference (Ding et al., 2003), the same transient problem of piezoelectric hollow cylinders no considering an axial magnetic field load is taken and the same

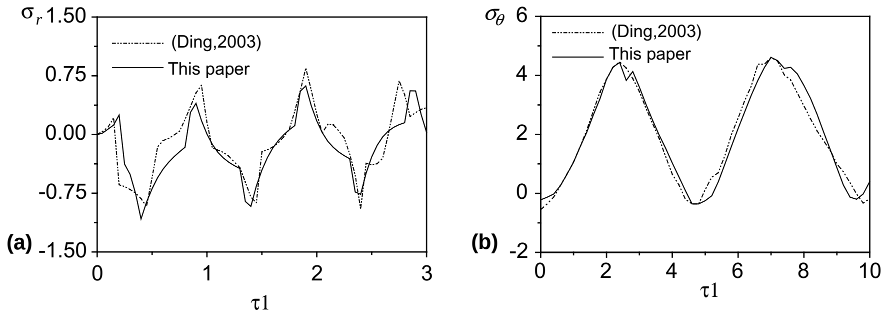

Fig. 5. Response histories of radial stress, σ_r, at $R = 0.5$ (a) and of hoop stress,σ_θ, at $R = 0$ in piezoelectric hollow cylinders no considering an axial magnetic field load, where $R = (r - a)/(b - a)$, $\tau 1 = C_{\mathrm{L}}t/(b - a)$.

parameters are taken: the ratio of internal radius to external radius $s = a/b = 1/2$, dimensionless time $\tau 1 = \frac{C_{\mathrm{L}}\tau}{(1-s)C_{\mathrm{V}}} = \frac{C_{\mathrm{L}}t}{b-a}$ and the dimensionless radial coordinate $R = \frac{\xi - s}{1-s} = \frac{r-a}{b-a}$. The boundary conditions are expressed in Eq. (48a,b). From Fig. 5a and b, one can see that the results from the two different methods are nearly the same.

5. Conclusions

(1) Although the transient responses of coupled fields have been studied by a number of authors, no published result can be used for a comparison with the present model. In fact, most of the previous works have focused on the transient responses of magnetothermoelasticity or electroelasticity. To our knowledge, no detailed report on the dynamic responses of piezoelectric hollow cylinders in an axial magnetic field is available in the literature. This is apparently due to the fact that the experiment on the dynamic responses of piezoelectric hollow cylinders in an axial magnetic field remains a formidable task.
(2) From the results and discussions in Example 1(a) it is seen that the results presented in Fig. 2a and b appears in the feature of the compression waves which is similar as the wave propagation in an infinite elastic solid with a cylindrical cavity subjected to only a sudden interior pressure (Achenbach, 1973 and Miklowitz, 1978). The other hand, in Example 3, the transient problem of piezoelectric hollow cylinders no considering an axial magnetic field load is calculated for simple comparison with reference (Ding et al., 2003). Therefore, it is concluded that the correctness of the numerical results in this paper is valid. Thus, the solving method may be used as a reference to solve other dynamic coupled problems in a piezoelectric hollow cylinder in an axial magnetic field, subjected to mechanical load and electric shocks.
(3) Because of the interaction between elastic deformation, electric field and magnetic field, a sudden mechanical load induces the response of electric displacement and electric potential, and the perturbation of magnetic field vector in a piezoelectric hollow cylinder. Likewise, a sudden electric potential also causes the dynamic stresses responses, and the perturbation of magnetic field vector in the piezoelectric hollow cylinder. Thus, applying a suitable electric excitation to a piezoelectric hollow cylinder can control the responses and distributions of dynamic stresses, and the perturbation of magnetic field vector in the piezoelectric hollow cylinder.
(4) Utilizing the knowledge of the response histories of dynamic stresses, electric displacements, electric potentials and perturbations of an axial magnetic field vector in a piezoelectric hollow cylinder, various electromagnetoelastic elements under mechanical loads and electric potential shocks can be designed to meet specific engineering requirements.

Acknowledgements

The authors wish to thank the supports of National Science Foundations of China (19972041, 10272075) and the referees for their valuable comments.

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